

Press Release

For immediate release

### Supplementary Note

#### 1. Quantum Hall Effect and Berry Curvature

The Quantum Hall effect (QHE) is the quantum mechanical version of the Hall effect. It can be generally divided into the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE). The integer quantum Hall effect was discovered by German physicist von Klitzing, who was awarded the Nobel Prize in Physics in 1985 for this discovery. The fractional quantum Hall effect was discovered by Chinese-American physicist Tsui, German physicist Störn, and American physicist Gossard. Based on this, Tsui and Störn shared the Nobel Prize in Physics with American physicist Robert Laughlin in 1998.

The phenomenon described by the integer quantum Hall effect is that when a two-dimensional electron gas is subjected to a transverse electric current, the longitudinal conductivity measured exhibits plateaus at multiples of the fundamental natural constant  $\frac{e^2}{h}$ . The integers appearing in the integer quantum Hall effect are topological quantum numbers. In mathematics and topology, they are known as Chern numbers (in honour of Chinese mathematician Shiing-Shen Chern), and they are intimately related to nontrivial Berry curvature in the band structure of solid materials. The quantum Hall effect and its derived phenomena, such as the quantum anomalous Hall effect, quantum spin Hall effect, and the recently discovered nonlinear Hall effect in twisted bilayer graphene, all exhibit novel and widely applicable electrical transport properties and the mathematical essence of nontrivial Berry curvatures.

#### 2. Hall Effects in Twisted Bilayer Graphene

Twisted bilayer graphene, as a representative of two-dimensional quantum moiré materials, offers the prominent advantage of easy controllability. By continuously scanning the twist angle, and applying electric and magnetic fields, one can finely tune the strength of interactions and electron filling in the system. This surpasses the limitations of traditional quantum Hall effect devices, such as two-dimensional electron gas materials. However, twisted bilayer graphene has a complex band structure (being a topological flat band system with long-range correlation effects), and factors like strain and inhomogeneities at the moiré scale, which are much larger than the lattice scale, need to be considered in model calculations. In this collaborative project, the theoretical team, via model design and large-scale computation, discovered that unlike typical topological materials, twisted graphene's unique topological flat bands allow for efficient control over the momentum-space distribution of Berry curvature. The experimental team indeed observed that, when a transverse driving current with a frequency of  $\omega$  was applied to the twisted graphene, a clear nonlinear voltage response in the longitudinal direction with a frequency of  $2\omega$ .

The success of this research has a potential impact on industries such as new materials and quantum information, particularly in applications involving frequency multiplication and rectification with low-frequency currents. One

can foresee that by controlling the Berry curvature dipole moments in twisted graphene, the terahertz detection with significant response and ultra-high sensitivity at room temperature now becomes possible.